Functional Analysis
Problem Set

V. M. Sholapurkar

1. Show that the space l_0 of complex sequences with at most finitely many non-zero terms is not closed in l^2.
2. Construct an infinite dimensional space with two non-equivalent norms on it.
3. In $C[0,1]$, consider the functional $F(x) = \int_0^1 x(t)f(t)dt$, where f is continuous. Find $\|F\|$.
4. Justify whether *true* or *false* : The closed unit ball in l^p is totally bounded.
5. Prove that the space l^∞ is not separable.
6. Show that the spaces c_0, c, l^1, l^∞ are not reflexive.
7. If X has finite measure and $p < q$, then prove that $L^q(X) \subset L^p(X)$.
8. If $f \in L^\infty$ is supported on a set of finite, then prove that $f \in L^p$ for $p < \infty$.
9. Justify whether *true* or *false*: For $f \in X^*, \|f\| = 1$, ther exists $x \in X$ such that $\|x\| = 1$ and $f(x) = 1$.
10. Let E be a measurable subset of $\mathbb{R}^n$. Show that it is not true that $m(E) = \sup\{m(U) : U \subset E, U \text{ open in } \mathbb{R}^n\}$.
11. Show that the Lebesgue measure is invariant under rotation and reflection.

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